[13.32] Show that every finite group **G** has a faithful representation in GL(*n*) where *n* is the order of **G.**

**Solution**

**Part A**. Show *T*(*G)* is a group representation

Proof of Part A is just an elaboration of Robin’s method, which is very slick.

Let **G** = {*g*1, …, *g*n}. A representation *T*(*G)* is the image of a group homomorphism . A homomorphism *T* is a function that preserves the group structure:

For all 

 is an invertible *n* x *n* matrix. I use Penrose’s hint to label the rows and columns of matrix  to indicate that the matrix takes  to  :

.

Matrix  can be written

,

matrix  can be written

 ,

and the product matrix is

.

A strategy to define *T* such that  is to put as many zeros as possible into the matrix so that the calculation becomes simpler. To that end, define

.

So,



and

.

Hence

,

, and

 .

The matriceshave precisely one 1 in every row and every column. The element  of the matrix  then becomes

.

That is,  ✔

**Part B**  Show *T* is faithful

*T* is faithful if it is one-to-one; i.e., if  So, suppose  



. ✔